172 Take Home Exam 2 Name: ______________________________

Fill in the definitions (look them up if needed)

1. A sequence a_n has a limit L if

2. A sequence is **increasing** if

3. A sequence is **decreasing** if

4. A sequence is **monotonic** if

5. A sequence is **bounded above** if

6. A sequence is **bounded below** if

7. A sequence is **bounded** if

8. Write the first 4 terms of the sequence
$a_n = \dfrac{n-1}{n+1}$

9. Write the first 4 terms of the sequence given by the recursion
$a_1 = 3, a_n = \dfrac{1}{4 - a_{n-1}}$
and find $\lim\limits_{x\to\infty} a_n$

10. Write the first 4 terms of the sequence given by the recursion
$a_1 = 2, a_n = \dfrac{1}{2}\left(a_{n-1} + \dfrac{5}{a_{n-1}}\right)$ and find
$\lim\limits_{x\to\infty} a_n$

11. Definition: A series $\sum_{n=1}^{\infty} a_n$ converges to S if

12. Find the sum of the geometric series $\sum_{n=0}^{\infty} \frac{4^n}{5^{n+1}}$

13. Find the following sum: $\sum_{n=3}^{\infty} \frac{3}{(n-2)(n+1)}$

14. Use the comparison test to test

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+3}}$$

for convergence. State explicitly what you compared it to.

15. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3}}$$

is conditionally convergent, absolutely convergent, or neither.

16. Use the ratio test to test $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$ for convergence.

17. Find the radius of convergence and the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$$

18. Find the Maclaurin series for

$f(x) = \dfrac{1}{(1-x)^2}$ and find the radius of convergence for that series.

19. Write the Maclaurin series for e^x

20. Write the Maclaurin series for $\sin(x)$

21. Write the Maclaurin series for $\cos(x)$

22. Write the Maclaurin series for x^3e^{-x}

23. Find the Taylor polynomial of degree 3 for $f(x) = \dfrac{1}{1+x}$ at $a = 2$

    Your answer should look like $c_0 + c_1(x-2) + c_2(x-2)^2 + c_3(x-2)^3$

24. From the answer above, write the Taylor series expansion for $\frac{1}{1+x}$ at $a = 2$

Your answer should look like $\sum_{n=0}^{\infty} c_n (x-2)^n$