Sample Final Name: ________________________

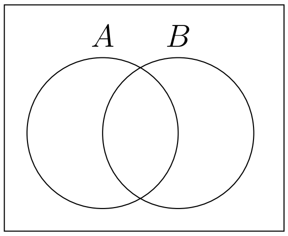
1. Write the converse, inverse and contrapostive of the compound statement: If I had a hi-fi, ma has a ham.
   (a) converse
   (b) inverse
   (c) contrapositive
2. Write a truth table for $(p \vee q) \to (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \wedge q) \to (p \vee q)$
T	T			
T	F			
F	T			
F	F			

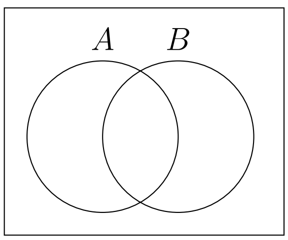
3. Prove $\neg(p \wedge q) \equiv \neg p \vee \neg q$ using a truth table.
4. Negate the statement"Everything is beautiful."
5. Negate the statement $\forall x P(x)$

6. Show $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$ using Venn diagrams.

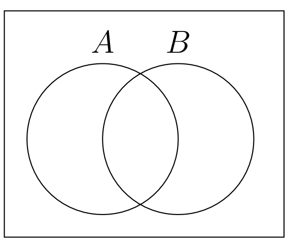
(a) $A \cup B$



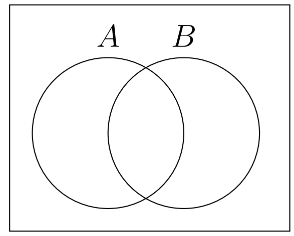
(b) $A \cap B$



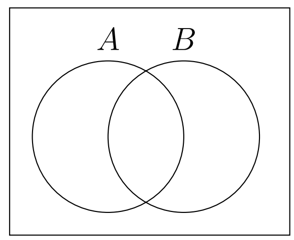
(c) $\overline{A}$



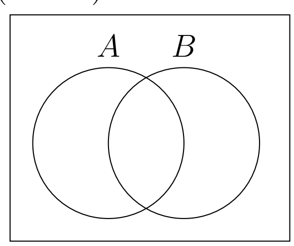
(d) $\overline{B}$



(e) $\overline{A} \cap \overline{B}$



(f) $\overline{(A \cup B)}$



Write the following statements in English, and negate each statment (in English). Determine whether the original statement is true, and give a justification. The domain of each statement is $\mathbb{N} = \{1, 2, 3, ...\}$

7. $\forall x \exists y (2x - y = 0)$

   (a) English

   (b) Negation

   (c) True or False

   (d) Justification

8. $\forall y \exists x (2x - y = 0)$

   (a) English

   (b) Negation

   (c) True or False

   (d) Justification

9. Definition: A function $f : A \to B$ is injective (one to one) if and only if

10. Negate the definition: $f : A \to B$ is not injective if and only if

11. Definition: A function $f : A \to B$ is surjective (onto) if and only if

12. Negate the definition: $f : A \to B$ is not surjective if and only if

13. Let $f : \mathbb{R}\backslash\{2\} \to \mathbb{R}\backslash\{0\}$ via $x \mapsto \frac{1}{x-2}$ i.e. $f(x) = \frac{1}{x-2}$

    (a) Show that f is injective.

    (b) Find an explicit formula for $f^{-1}(x)$

14. Prove the (rather obvious) fact that if n is an integer and $3n-2$ is even, then n is even

    (a) by contraposition

    (b) by contradiction

15. Find the first 5 terms of the sequence defined by the recurrence relation

$$a_n = a_{n-1} + n, a_1 = 1$$

16. Show that $a_n = 2^n - 1$ is a solution to the recurrence relation

$$a_n = 2a_{n-1} + 1,$$

17. Find $\sum_{k=1}^{100} 2k+5$

18. Find $\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k$

19. Prove $\sum_{j=1}^{n} 2j+5 = n^2+6n$ by induction

   (a) Base step:

   (b) Induction hypothesis:

   (c) Inductive step:

20. Prove that $n^2 + n$ is even by induction:

    (a) Base step:

    (b) Induction hypothesis:

    (c) Inductive step:

21. Prove $\forall n \in \mathbb{N}, 5|11^n - 6^n$ that is $11^n - 6^n$ is divisible by 5

    (a) Base step:

    (b) Induction hypothesis:

    (c) Inductive step: Hint $11^{k+1} - 6^{k+1} = 11^{k+1} - 11 \times 6^k + 5 \times 6^k$

22. Factor 392 and 420

23. Find $lcm(420, 392)$

24. It is fairly clear that $gcd(101, 23) = 1$ since both are prime. Using the Euclidean algorithm find the Bezout coefficents, that is, find s and t so that $101s + 23t = 1$

25. What is 101 modulo 23?

26. Find the inverse of 4 modulo 9

27. Solve $4x \equiv 3(\bmod 9)$

28. Solve the system

$$x \equiv (2 \bmod 3), x \equiv (1 \bmod 5\ ), x \equiv 2(\bmod 7)$$

29. Show that if there are 30 students in a class, at least two must have last names that begin with the same letter.

30. Show that among any group of 6 integers, there is a pair x, y with $x \equiv y(\bmod 5)$

31. How many cards must be drawn from a deck to ensure that there are either 4 clubs, 3 hearts, 2 spades or 1 diamond selected?

32. Verify Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

for $n = 6, k = 4$

33. Let A be the set $\{1, 2, 3, 4, 5\}$ and R be the relation $R = \{(x, y) | x \text{ divides } y\}$

   (a) Write R as a set.
   (b) Draw a diagram for R
   (c) IsR reflexive? Be explicit.
   (d) Is R symmetric?
   (e) Transitive?

34. The sets $\{a, b\}, \{c, d\}\{e\}$ form a partition of $S = \{a, b, c, d, e\}$ Diagram the corresponding equivalence relation.

35. Let $A = \{0, 1, 2, 3, 4, 5, 6\}$and R be the relation $R = \{(x, y) | x \equiv y \bmod 3\}$

   (a) Show explicitly that R is and equivalence relation.
   (b) What are the equivalence classes of R?